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NONSYMMETRIC BUCKLE PATTERNS IN PROGRESSIVE PLASTIC BUCKLING

By

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September 1971

EUSTIS DIRECTORATE
U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
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13. ABSTRACT

This report presents the results of a series of tests made on shells which buckled plastically. It is shown that nonsymmetric patterns can be developed. These appear to be nonextensional in character. They are very similar to the Yoshimura pattern which occurs as the bending case for thin shells.

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The data contained in this report are the result of research conducted on the progressive plastic buckling of cylindrical shells under axial compressive load. The method of buckle generation and development in the thick-walled shell and the influence of geometric and mechanical properties were studied.

The report has been reviewed by this Directorate and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

This program was conducted under the technical management of Mr. James P. Waller, Structures Division.

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SUMMARY

This report presents the results of a series of experiments on the progressive plastic buckling of cylindrical shells under axial compressive load. It shows that, for shell bodies of an R/t less than 100, the normal axisymmetric ring buckling will develop into nonsymmetric patterns. It is demonstrated that there exists also a class of shells within this thickness radius range for which nonsymmetric plastic buckling always occurs without the prior formation of a ring. It appears from the limited number of tests made that, for a particular R, R/t, material and rate of loading, there is a critical value of L, above which there is a high probability of the buckle pattern developing in a nonsymmetric fashion. It seems probable, too, that there are bands of R/t for a particular L/R, R, material and rate of loading for which the buckle number will be constant. The experiments tend to indicate that the postbuckling efficiency of the shell decreases with increasing buckle number.

The nonsymmetric patterns demonstrated appear to be inextensional deformations. They are very similar in character to the Yoshimura pattern which occurs as the limiting case for thin shells in axial compression and under impact loading. Load-displacement histories are presented for some of the various modes of failure demonstrated.

FOREWORD

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INTRODUCTION

The unstiffened circular cylindrical shell is a common component in engineering. Generally, stability rather than specific strength is the criterion on which it is designed. Unfortunately, despite tremendous concentration of research effort, both theoretical and experimental, the disparity between predicted and observed behavior is so great that the subject must still be considered an open field.

The latest experimental data[2] tend to show that the initial buckling load for a perfect elastic specimen would be in full accord with the classic value predicted by small-displacement-theory considerations. This value, of course, would not be inconsistent with the tenets of large-displacement theory. However, it has recently been demonstrated that machine stiffness in no way influences the initial buckle load for an imperfect shell[3] - a finding which throws considerable doubt upon the validity of some of the existing large-displacement developments.[4]

There is little question that initial imperfections[5,6] play an important part in the stability process, and it is likewise clear that boundary conditions are significant.[7,8,9] Nevertheless, it seems unlikely that these two conditions are the only fundamental mechanisms which exercise control in the buckling process.

In a recent theoretical study, Mayers and Rehfield[10] reexamined the buckling behavior of unstiffened, circular cylindrical shells. They concluded their work with a most significant remark: "Little improvement toward a satisfactory solution of the problem can be gained from any theory in which the strain-displacement formulation does not take cognizance of the size of rotations to be expected in a realistic situation and the physical law between stress and strain does not account for inelastic behavior."

This analytical opinion is in excellent accord with observations made by Schnell[11] as the result of careful experimental studies. They support entirely the contentions of Horton and Durham[12] made as a result of a series of repeated load tests on cylinders and cones. The work of these latter researchers showed that there is some effect in the region of the buckle which influences the behavior of the system. Subsequent research[13] showed that the effect was severely dependent upon the depth to which the buckle was permitted to develop.

Thus, we conclude that plasticity may be an important factor in the buckling and postbuckling behavior of axially compressed cylindrical shells.

Generally speaking, buckling which is regarded as plastic has normally been seen as ring buckling. Elastic instability of thin-walled shells is likewise characterized by diamond patterns. The photographs of Figures 1 and 2 are good examples of the elastic and plastic buckle systems,

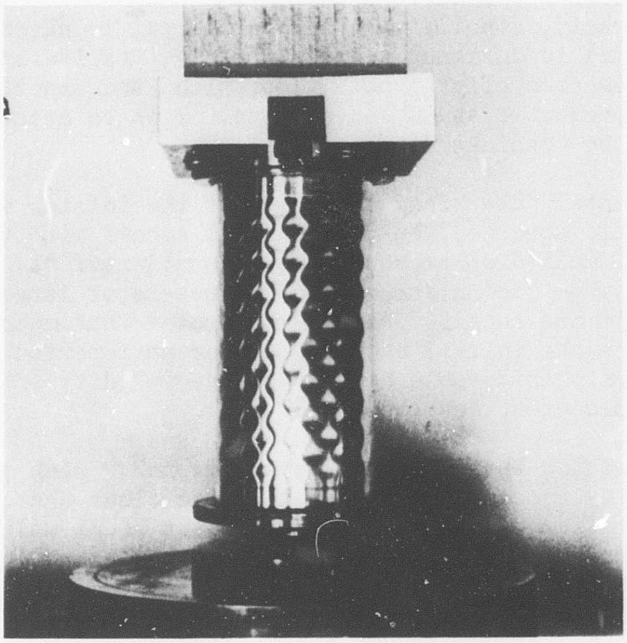


Figure 1. Photograph of Cylinder With Complete Developed Buckle Pattern.

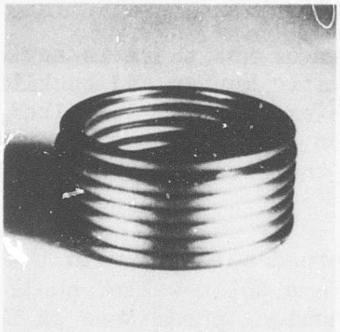


Figure 2. Plastic Ring Buckles.

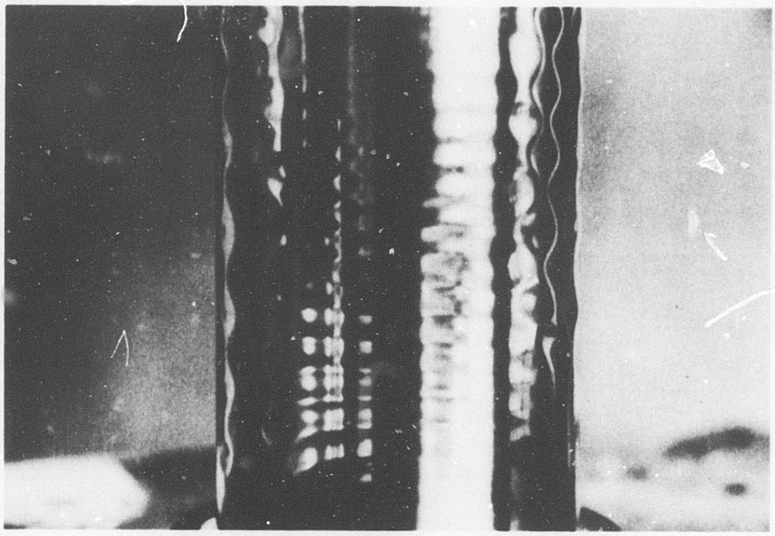


Figure 3. Unusual Combination Pattern.

respectively. However, as the photograph of Figure 3 shows, there can be unusual cases in which the two systems are simultaneously present.

We are led, then, to ask whether or not there is any connection between buckling which begins in an elastic manner and buckling which essentially is plastic from the onset. The experiments described in this report were made as part of a basic study on the question. They constitute a small section of current research on the effects of plasticity in the buckling of shell bodies.

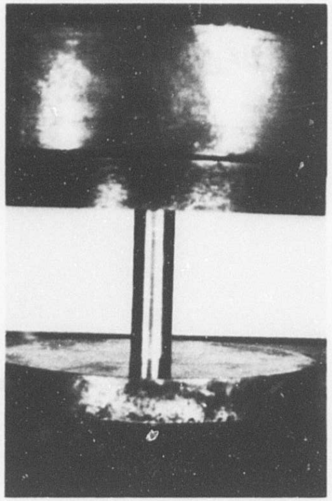
In these studies, we were concerned not only with the method in which a buckle generates and develops in a thick-walled shell, but also with the influence of geometric and mechanical parameters on this process. Thus, our tests have been made on a wide range of materials - variation in length/diameter, thickness/diameter and rate of loading has been considered. The load-displacement behavior has been recorded, also, in several cases of interest.

DISCUSSION OF THE PROBLEM

Let us consider a cylindrical shell positioned between the platens of a testing machine. When such a shell is loaded in compression, the length of the cylinder decreases but the diameter increases due to the Poisson expansion. The increase in diameter is, however, prevented at the edges of the shell because of the friction which exists between the ends of the test vehicle and the loading plates. This local variation in geometry causes the direct load to induce bending actions at the ends of the specimen. The result is that the generators of the cylinder are deformed.

The question of importance is whether the bending stresses at the edges or a possible instability will determine the strength of the shell. It is well known that there is a class of shells of low R/t value for which the bending stresses grow beyond bounds when the applied load reaches a critical value connected with a ring-shaped buckling pattern. When this occurs, the first and largest bulge on the cylinder is squeezed flat. Progressive stages in this failure mode are as shown in the photographs of Figure 4. This result is not new; it was reported by Geckeler[14] in 1928 and is described in detail in textbooks on elastic stability of plates and shells.[15]

More recent literature[16,17] on this topic treats only the same axisymmetric distortion. Indeed, it is generally implied that the large-displacement inextensional type of deformation is unlikely since the forces needed to develop such a system are predicted to be greater than those required to produce the axisymmetric ring systems. There is clear fallacy in such argument. It cannot be denied that, for very thick shells, the axisymmetric pattern always occurs. It is likewise irrefutable that, for very thin-walled sections, the diamond shape is common. There must be some intermediate geometry at which transformation from one system to the other can take place - a state, perhaps, in which there is an equal likelihood of either pattern developing. Knowledge of this area should be significant to the understanding of the buckling process for thin-walled shells.



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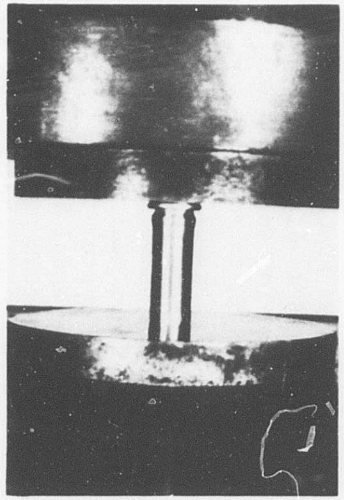

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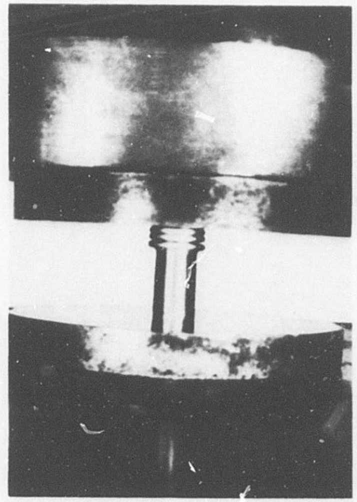

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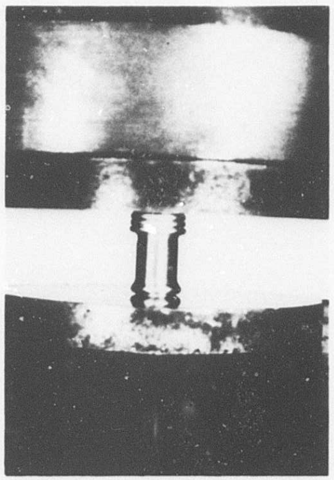

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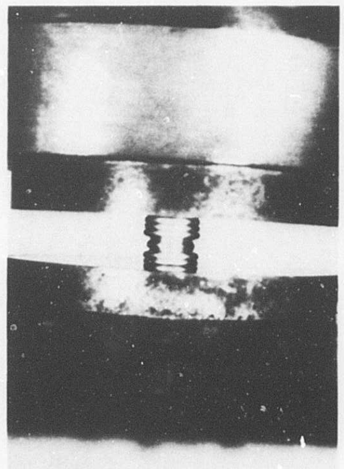


(e)

Figure 4. Development of Ring Buckles.

(a) The unbuckled tube.

(b) Complete flattening of first ring. Note: Second ring beginning to form.

(c) Third ring developed.

(d) Ring buckles starting at the other end of shell.

(e) Rings well developed at both ends and central ring now forming.

EXPERIMENTAL PROGRAM

NATURE OF THE TEST SPECIMENS

The test specimens were uniform-thickness, right-circular cylindrical shells of either aluminum, brass, copper or stainless steel with accurately squared ends. A wide range of thickness-to-diameter and length-to-diameter ratios was used to obtain the results given. The majority of tests were made with tubes in the as-received condition but, for certain tests in which R/t variation was considered, the test family was manufactured by turning from a common tube stock.

TEST SETUP

The specimens were set in a Baldwin-Lima-Hamilton 60,000-lb-capacity universal testing machine. Ground steel plates were interposed between the platens and the test specimen. In those cases for which load-displacement histories were taken, a load transducer was placed between the lower platen and the lower steel plate. Great care was exercised to ensure that the platen motion was vertical.

TEST PROCEDURE

Tests were made under various loading conditions. Some tests were run at high rates, some at low rates, and some with rate variations during the test. In all cases, however, the loading was pure axial compression. Where appropriate, loads were determined at every 1/1000-inch end shortening.

BUCKLING MODES

We have found that, from a stability point of view, there are essentially five classes of cylindrical shells when the loading action is uniform axial compression:

1. Those in which the initial buckle is plastic and axisymmetric and remains so with subsequent loading.

2. Those in which the initial buckle is plastic and axisymmetric but develops into a nonsymmetric pattern of elliptic, rectangular, triangular or square cross section with subsequent loading. In this class, there are essentially two subclasses: (a) those in which the nonsymmetry occurs immediately after the formation of the first ring, and (b) those in which a number of rings are formed prior to the development of the nonsymmetric pattern.

3. Those in which the initial buckle is plastic and is always nonsymmetric in character.

4. Those in which the initial buckle is elastoplastic in character, is nonsymmetric in form and becomes fully plastic with increasing load.

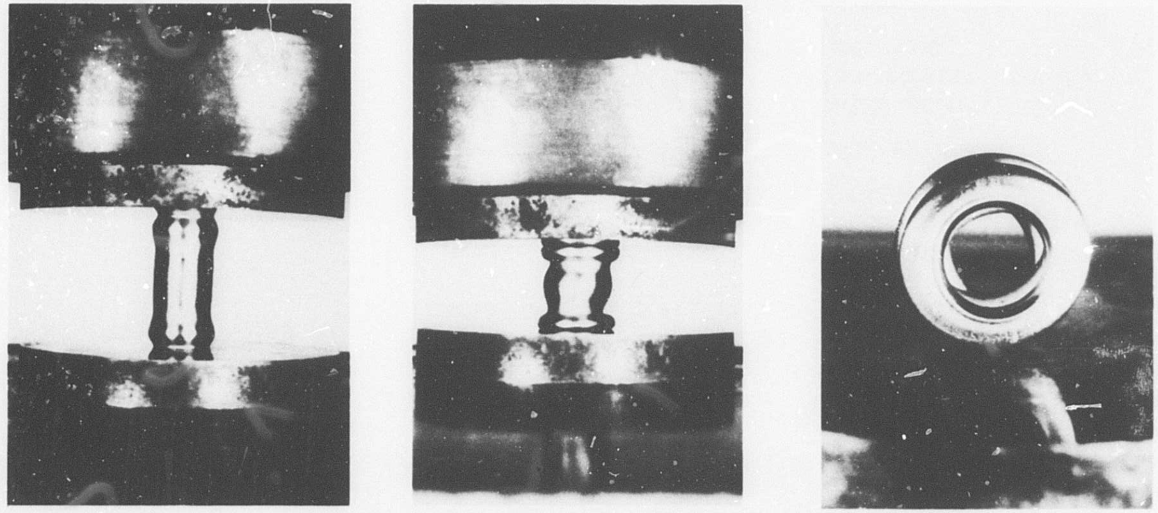
5. Those in which the initial buckle is elastic in character, is nonsymmetric in form, which becomes fully plastic with increasing load.

In this report, we deal with cylindrical shells which belong to the first three classes. The development of these several shapes is interesting and is portrayed in the various photographs which are given in the report.

In Figure 4 we show the progressive development of ring buckles. These pictures were taken during the buckling of a steel tube. In Figure 4(a), the initial tube is seen. The first ring buckle forms at the top of the specimen. In Figure 4(b), the first ring has flattened and the second ring is beginning to form. By the time the condition of 4(c) is reached, the third ring has developed. The next stage of the development takes place in Figure 4(d) when rings are seen to be forming at the lower end of the specimen. This is developed and, by the time the displacement reaches the value portrayed in 4(e), the central ring is beginning to grow quickly. The ultimate developed ring pattern is that shown already in Figure 2.

In Figure 5(a-c), we see successive stages in the formation of an intermediate pattern. Here there is definitely a pronounced tendency for the specimen to buckle in a two-lobe type of failure, but the two lobes have not fully developed; instead, the cross section has become elliptical. It is interesting to note that the ellipse in the first layer is orthogonal to the ellipse in the second layer and parallel to the ellipse in the third.

In Figure 6(a-d), we show successive stages in the development of a two-lobe type of failure. This is a very interesting sequence. We notice that, as the load increases, a ring begins to form at the upper end of the cylinder. This ring develops to a high degree, and then the shell begins to depart from circularity immediately below the ring. A fold is produced; the material flows into this plastic hinge line and the cross section of the cylinder alters appreciably. The two creases develop, flattening as they do so. Next, in a plane normal to these folds, there appear inward triangular indentations which progressively increase in size as the load increases, continulously folding their upper ends inward. As the folding develops further, triangular indentations develop and the process of folding, indenting and developing continues until the shell has deformed as shown in the final picture. The ultimate buckle pattern consists of two-lobe layers in which the lobe direction of each layer is at $\pi/2$ radians to the one preceding. It is significant to note that the first stage in the deformation process was the formation of a ring, and this ring changed its shape under the loading action until it became a two-sided figure. In some cases, for example, those shown in Figure 7, a slightly different action occurs. A number of rings are produced and remain essentially circular but, as the overall length of the specimen decreases, the tendency to progressively buckle in rings decreases and the two-lobe failure takes place. Generally speaking, when this happens we do not get a very pronounced two-sided figure such as portrayed in Figure 6, but the sections distort from circles to essentially ellipses.



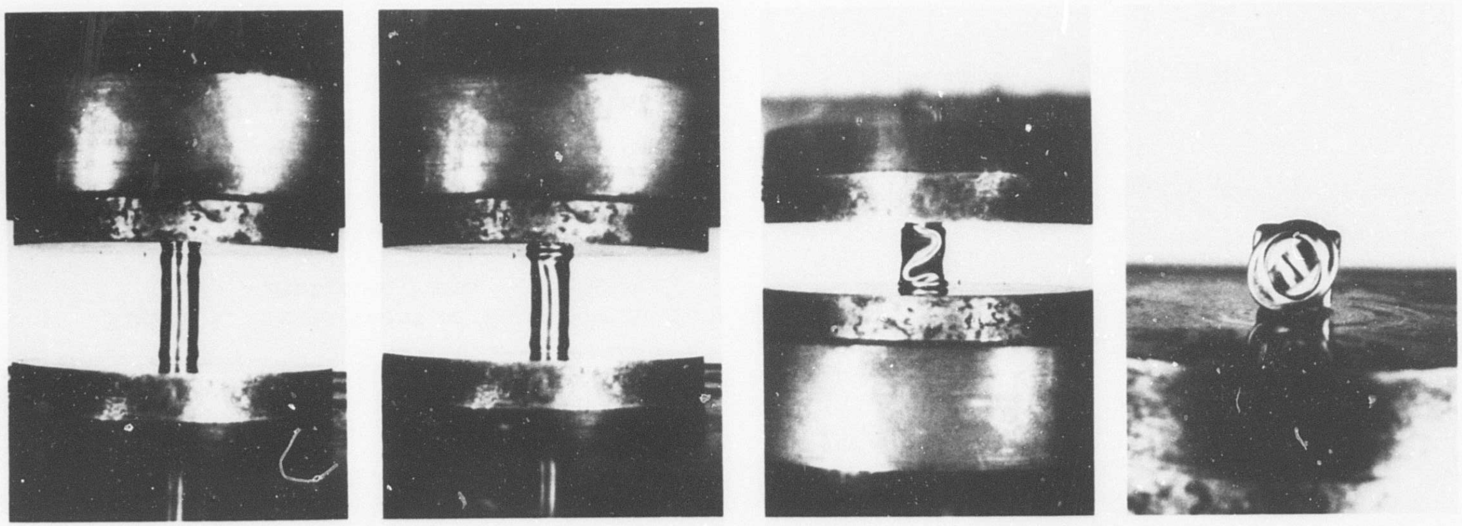
(a) (b) (c)

Figure 5. Development of Nonsymmetric Pattern of Rotated Elliptic Cross-Sectional Form.

(a) First stage in the development of the buckles;buckles are appearing to the same degree at both ends.

(b) The section beginning to "twist" and change shape from circular to elliptical.

(c) End view showing the elliptic cross sections with layer-by-layer rotations.



(a) (b) (c) (d)

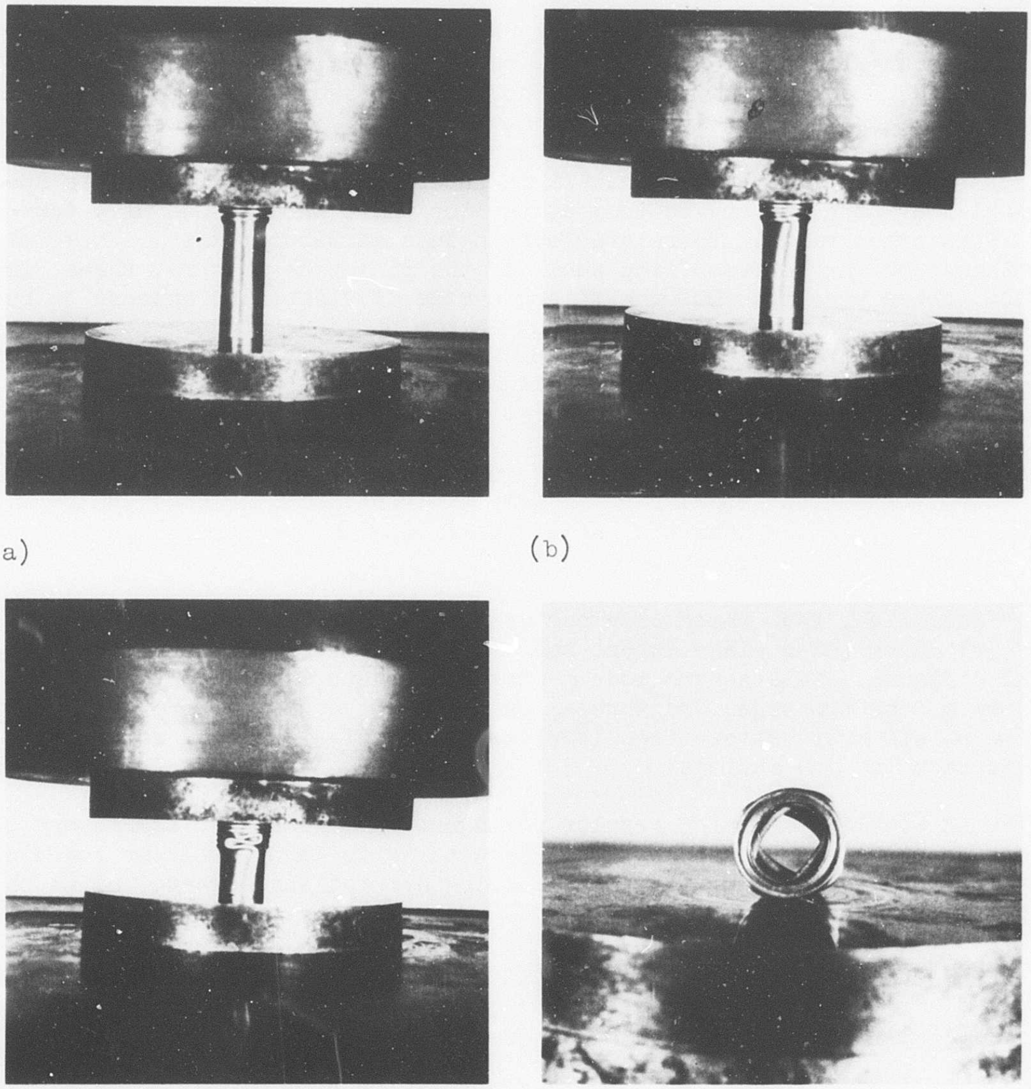
Figure 6. Development of a Nonsymmetric Two-Lobe Pattern of Rotated Rectangular Cross-Sectional Form.

(a) Development of ring buckling. Note: This specimen clearly shows evidence of a tendency to produce a number of ring failures.

(b) Inward motion of the wall has now started.

(c) Wall motion occuring at a third point. Note: This motion is in a direction parallel to the initial but normal to the preceding.

(d) End view of this buckled cylinder.



(a) (b) (c) (d)

Figure 7. Development of a Symmetric-Nonsymmetric Pattern Formation.
(a) Copper tube in which an end-ring buckle has started and is beginning to develop into a two-ring pattern.
(b) The change from ring deformation to two-lobe failure is clearly seen in this picture.
(c) Inward motion of wall developed in a direction normal to the previous.
(d) End view showing both ring and elliptic sections.

If the geometric and mechanical properties are suitable, the three-lobe failure can also be found. This is shown very clearly in the photographs of Figures 8 and 9. In the first photograph, Figure 8, we see the triangular pattern at an early stage of development. In this case, a ring was formed but it did not flatten; instead, it changed directly to the triangle shape shown in the photograph. It is interesting to note that this specimen buckled at both top and bottom; at the top there is evidence of a tendency to form other rings. There are four in this particular case. There also is evidence at the bottom of the shell of the ring-type pattern, three rings in fact, being present. But the triangle mode of distortion appears to require less energy for its development; so, as the buckling process continues, the rings disappear and triangular shapes take their place. The deformation pattern is rotated through $\pi/3$ radians as we move from one level to another. This is very clearly seen in the view given in Figure 8(b).

In Figure 9(a-d), we show progressive stages in the development of such a pattern. The great regularity is very apparent. Three-lobe-type failures can occur after the formation of a single ring or may occur after a number of rings have been produced.

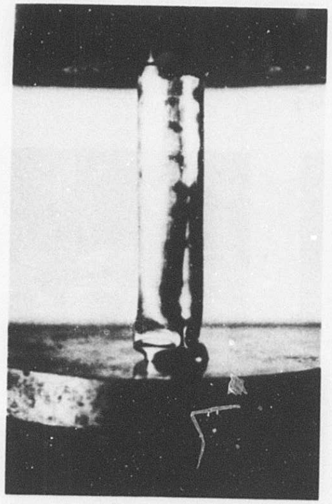
We find, also, that there is a particular geometry for which a similar type of procedure takes place except that the nonsymmetric pattern is a four-sided figure. This square mode portrays characteristics very similar to those of the two-sided and three-sided buckle systems. There is a progressive rotation of pattern from layer to layer. The angle of rotation in this case was $\pi/4$ radians.

Plastic buckling in which regular pentagons, hexagons, heptagons, and octagons occur is also possible. The characteristic layer-by-layer rotation is still present, in general the angle being π/n radians - where n is the number of lobes.

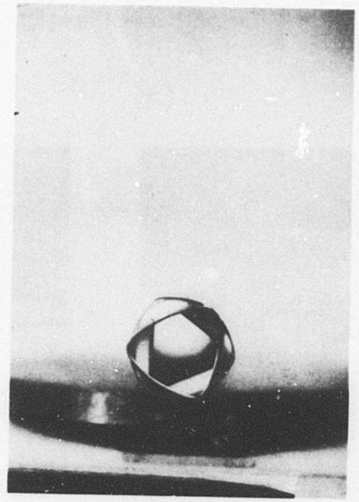
DISCUSSION OF THE NONSYMMETRIC SHAPES

Geometry of the Shapes

When we examine in some detail the shapes which are formed in the various buckle patterns described in the section on "Buckling Modes", we notice a most significant fact. In all cases of axisymmetric deformation, the perimeter of the shell is appreciably altered in length both at the inside and outside of the folds. This is scarcely surprising, since it is in these regions that there are very high strains and great material flow results. In the nonsymmetric patterns, the situation is quite different. In these cases, the perimeter of the shell along a horizontal fold line very closely agrees with the value in the undistorted shell. Closer examination of the patterns leads to the conclusion that there is a great similarity between these and the inextensional deformation patterns previously described by Yoshimura.[18] It is difficult, of course, to visualize this inextensional deformation for the two-lobe failure such as that shown in Figure 6. The fact is, however, that the two-lobe-type failure bears the same relationship

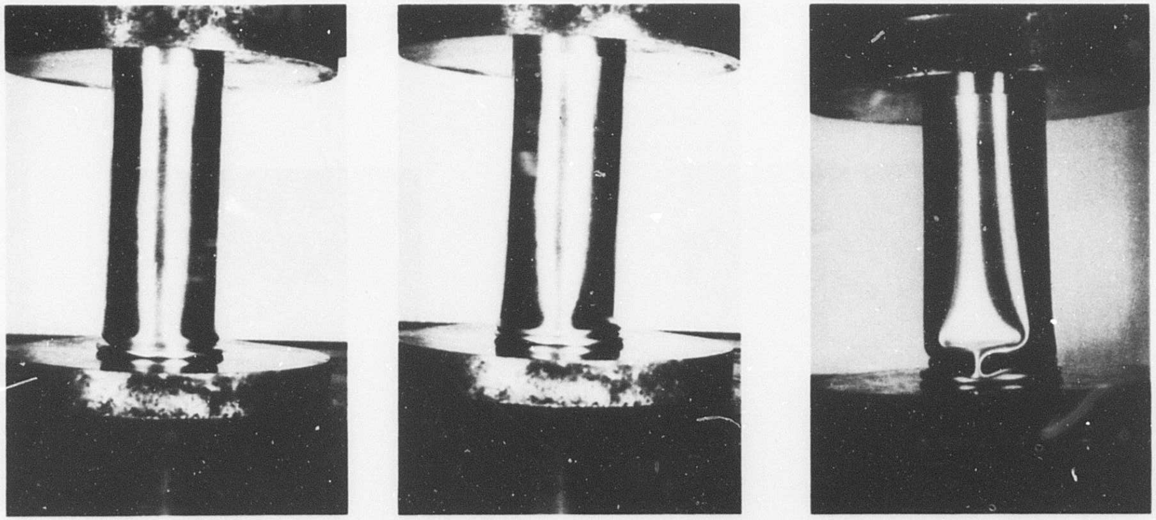


(a)



(b)

Figure 8. A Triangular Cross-Section Mode.
(a) The first stage was the formation of a ring, which changed to the triangular form without flattening. Note: Evidence of many rings beginning to form. The tube used here was of stainless steel 0.75-in. OD, and 0.020-in. thickness and 3-3/8-in. length.
(b) End view of specimen. Note: the characteristic layer-by-layer rotation is very evident.



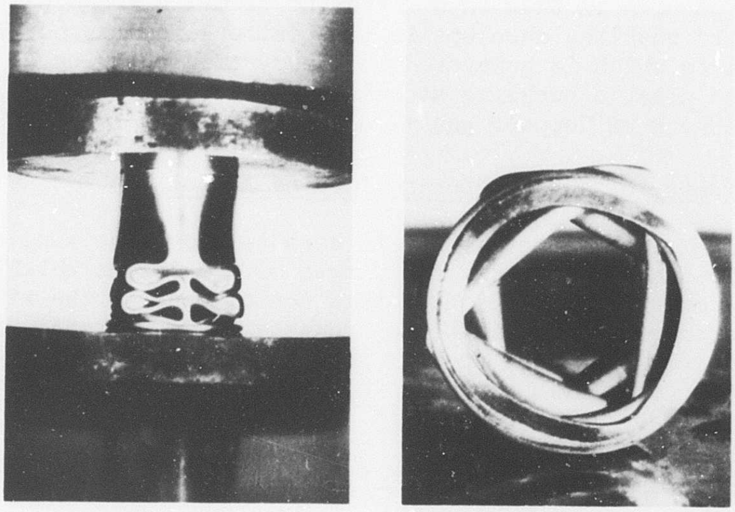
(a) (b) (c)

Figure 9. Development of Nonsymmetric Pattern Triangular Form.

(a) Brass tube 1.5-in. OD, 0.028-in. wall thickness, 5-3/8-in. length. First ring buckle forming.

(b) The second ring has developed completely. Inward motion of the wall is now very apparent. Note: There is clear evidence of a tendency for the wall to distort in such a manner that the local cross section is triangular.

(c) One triangular layer is fully formed, the second is well developed. Notice the rotation.



(d) (e)

Figure 9. Development of Nonsymmetric Pattern Triangular Form (Continued).
(d) Fourth fold developed. Fifth starting.
(e) View showing triangular cross sections in the folds rotated layer by layer.

to the Yoshimura pattern as the other types. This is clearly evidenced in Figure 10, which is a rather special case. It is a failure which began at the center of the tube. Here, there is a clear fold and a distinct two-lobe failure.

The Yoshimura pattern is that into which the normal diamond pattern experienced in elastic buckling develops in the postbuckling range. Furthermore, it is the pattern which is generated in the dynamic buckling of shells and the progressive plastic buckling at relatively low strain rates. This is clear from the work of Coppa[19] and from the photograph of Figure 11.

Changes in Geometry of Buckles During Loading Action

Under "Buckling Modes" we noted that in certain cases the ring buckles are formed to a limited degree along the length of a specimen which subsequently buckles into a multilobe form. When this action occurs, the rings previously formed appear to blend into the inextensional deformation smoothly. In a limited number of cases, another peculiarity can be noticed. Shells which deform into a multilobe pattern of high n may, on continued loading, change to a lower value of n.

Influence of Shell Geometry on Buckle Behavior

Experiments have shown that for a particular R, R/t material, and rate of loading, there exists some L/R below which the probability is high that buckling will be of the first class and above which it is equally likely that the behavior will be consistent with the second class. This critical length is clearly seen from the graphs of Figure 12.

In a like manner, it appears from experiments that there are almost definite bounds to the value of R/t for a prescribed n (number of buckles) when the L/R, R, material, and rate of loading are kept constant (Figure 13).

When the load-displacement histories for shells of varying buckle number, obtained by keeping L/R, R, material, and rate of loading constant, are plotted in a nondimensional form, as in Figure 14, we note that with the higher number of "lobes" there is a clear tendency for "snap" to occur. We observe, too, that the postbuckling efficiency reduces as n increases (Figure 15). We recall that n increases as R/t increases (Figure 12), and thus we reiterate the accepted fact that the postbuckling efficiency tends to a lower bound, a bound which may well be influenced by plastic deformations as the ratio of radius to wall thickness increases.

The observations with regard to the number of buckles and the relationship between n and R/t seem fully consistent with the findings of Wilson and Newmark,[20] who record in their research a low number of buckles for plasticity-buckled shells but who do not show the nature and character of these buckles.

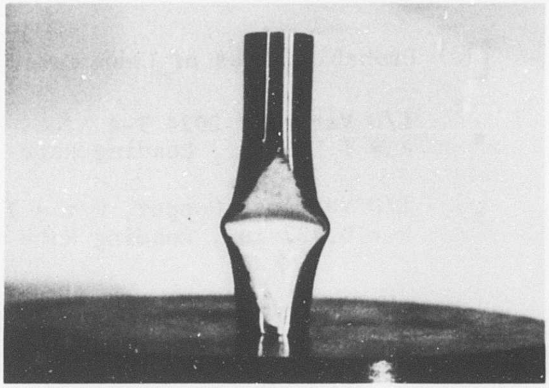


Figure 10. A Two-Lobe Failure Which Began at the Center of the Tube.

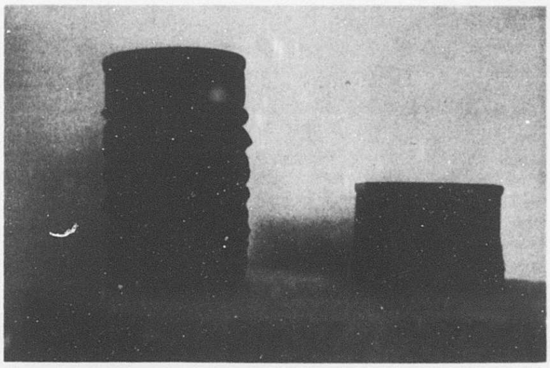


Figure 11. A Plastic Buckle Pattern Produced at Low Strain Rate Which Has the Characteristics Normally Seen in Dynamic Buckling.

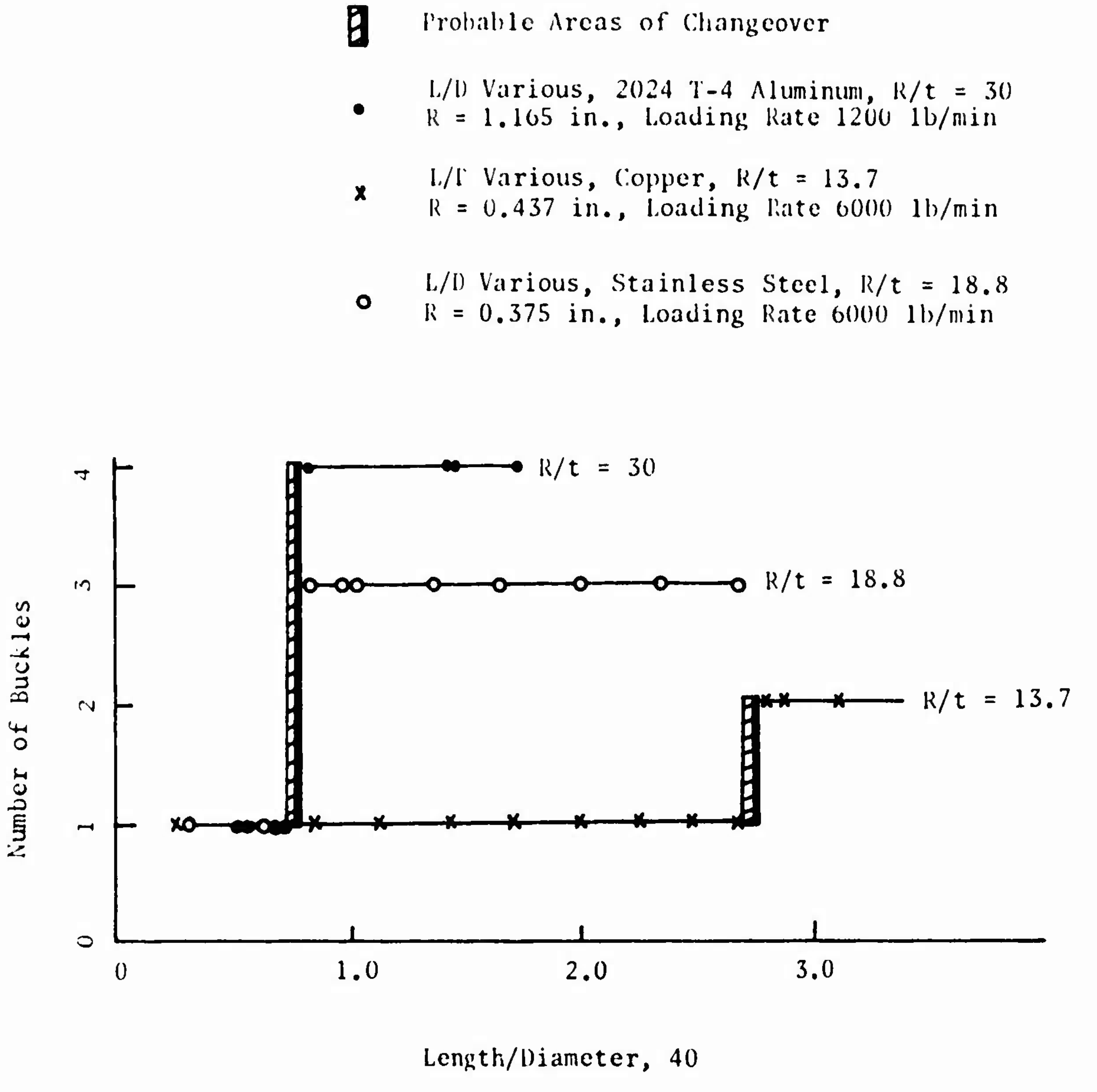


Figure 12. Variation in Buckle Number as a Function of L/D for a Constant R, R/t, Material and Rate of Loading.

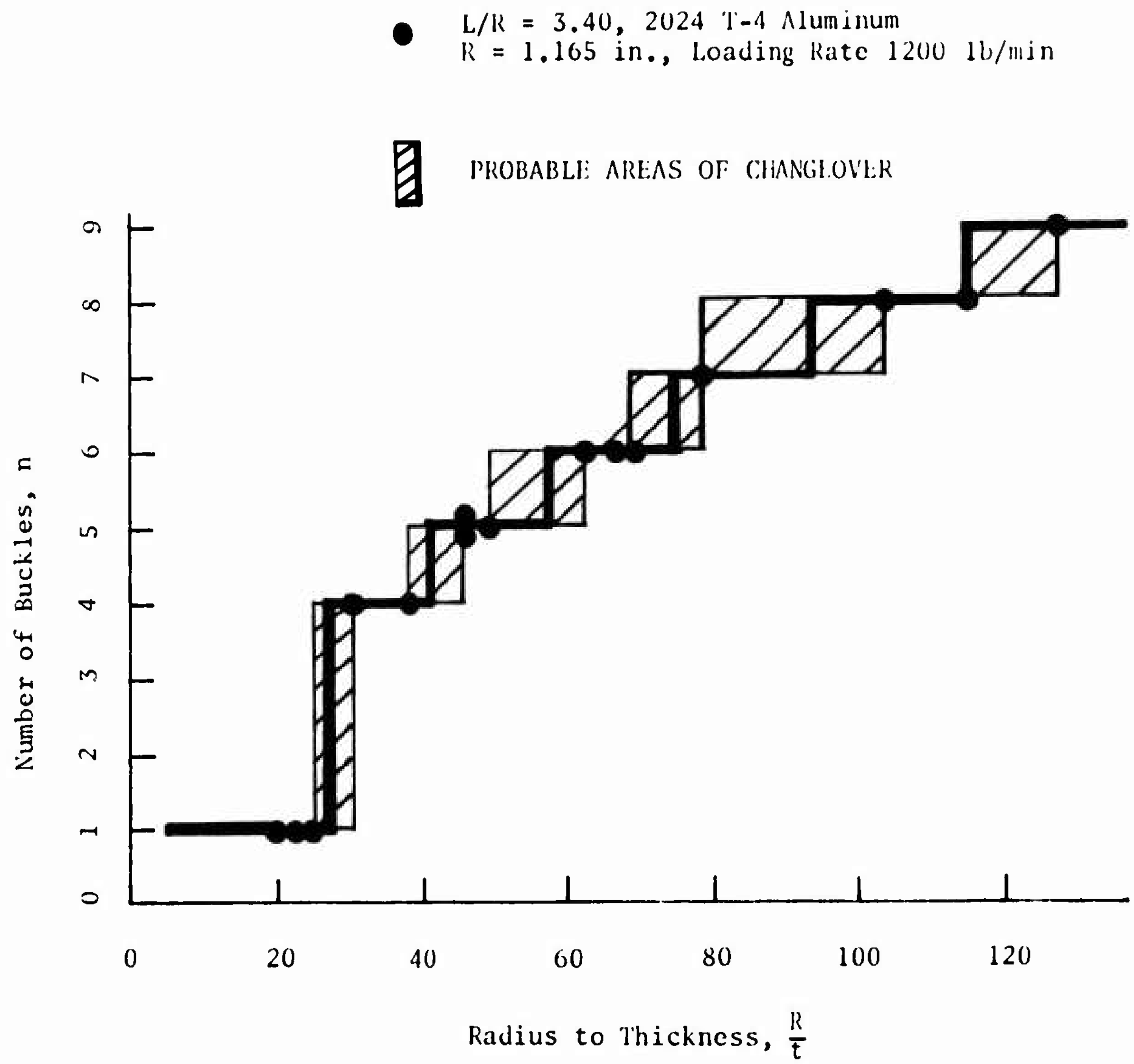


Figure 13. Variation in Buckle Number as a Function of R/t for a Constant R, L/R, Material and Rate of Loading.

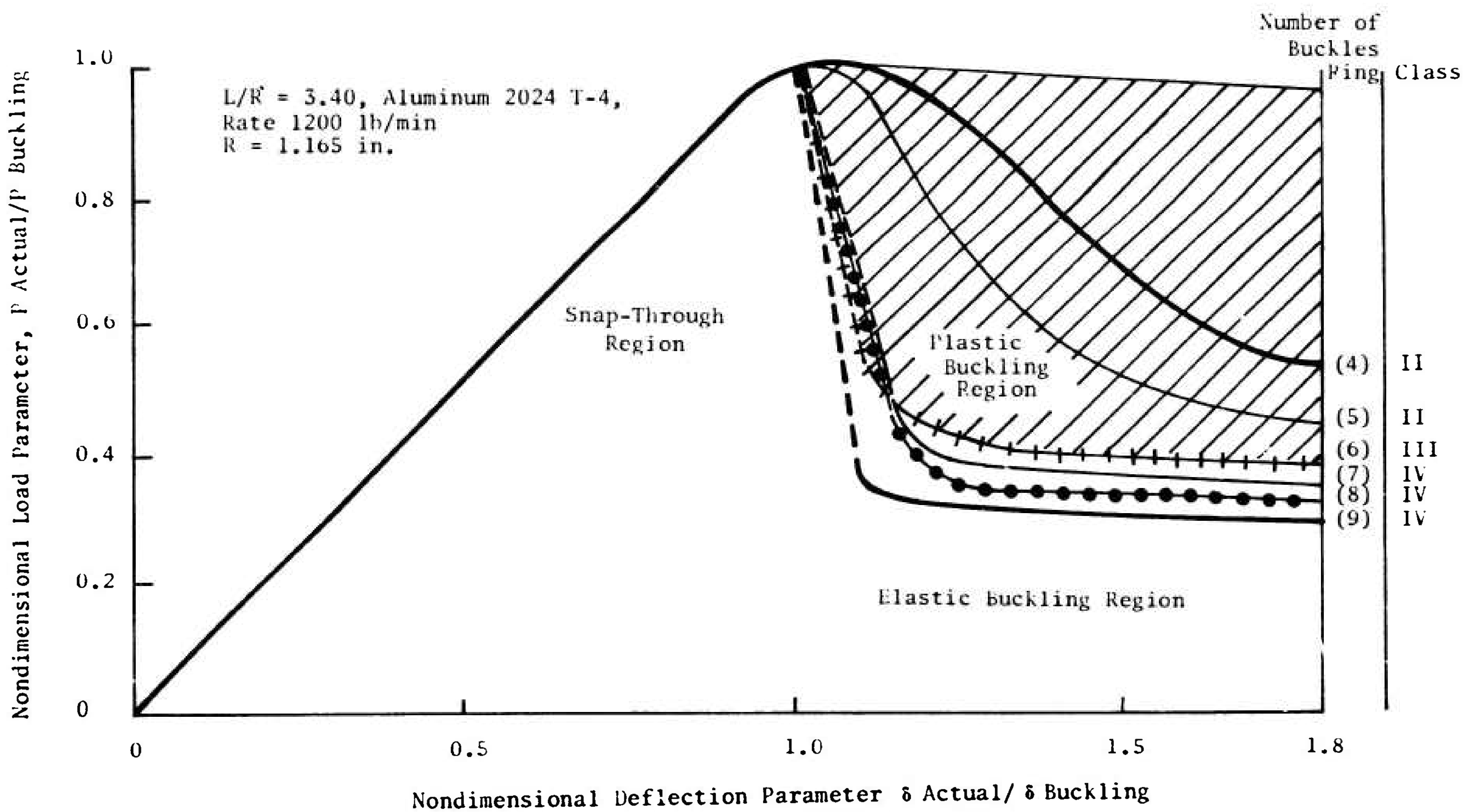


Figure 14. Nondimensional Plot of Load-Displacement for Various Number of Buckles.

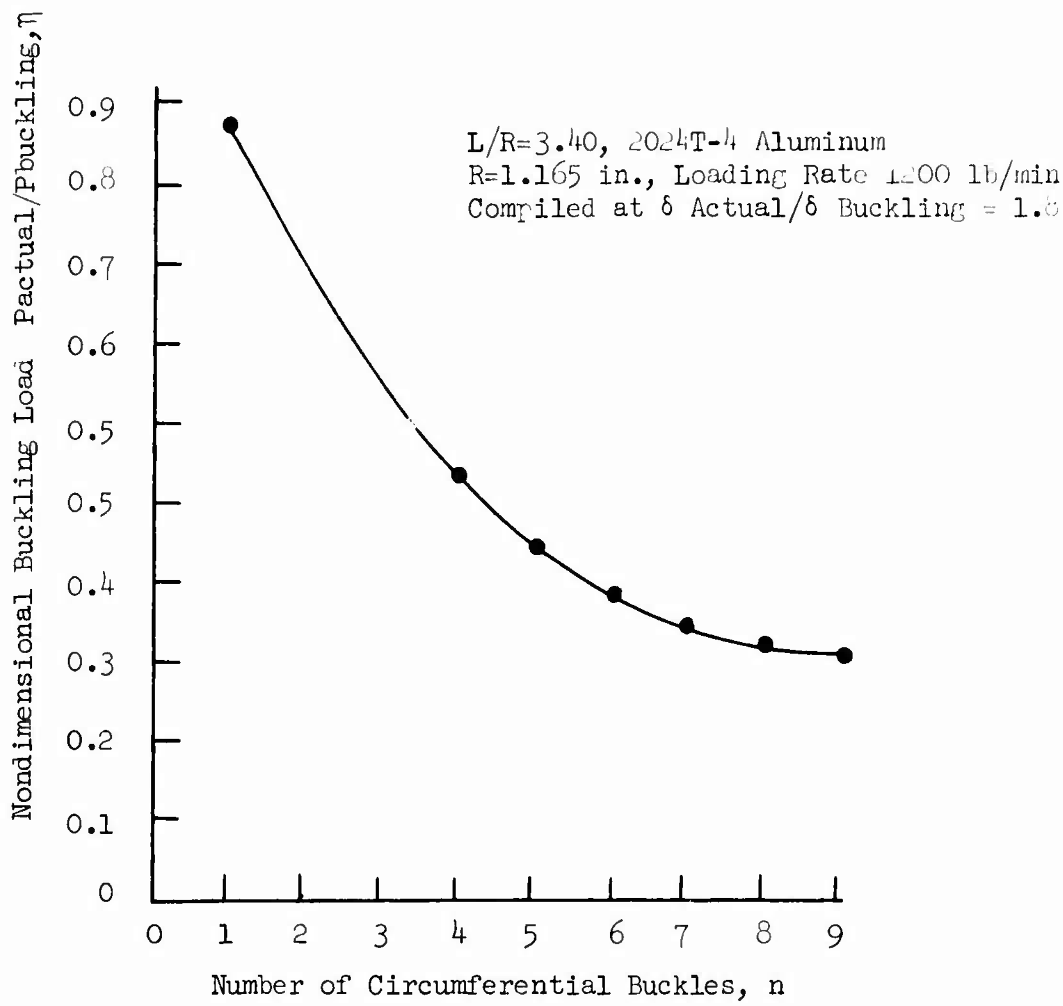


Figure 15. Postbuckling Efficiency as a Function of Buckle Number.

LOAD DISPLACEMENT HISTORIES FOR CLASS I AND CLASS II BUCKLING

When the tests described in the section of "Buckling Modes" were carried out, the load displacement history for various failure modes was determined. The load displacement for a typical ring-buckle pattern is presented in Figure 16. Initially, the body behaves elastically, and the testing machine load rises at a steady rate. Yielding begins and the rate of load development falls; the load rises to a maximum value, but as the instability begins, it falls off rapidly until the ring develops to a maximum diameter. There are successive rises and falls in load level as the various plastic regions form and develop. This cycle of events repeats with great regularity of form. The maximum and minimum loads are strikingly consistent, as is the shape of the repetitive portion of the load-displacement curve.

In cases of nonsymmetric buckling, the two-and-three-lobe failures also are given in Figure 16. There is clear oscillatory character in the load-displacement relationships, but there is not, by any means, the symmetry which characterized the ring system. The pattern of buckling, however was very consistent in both cases.

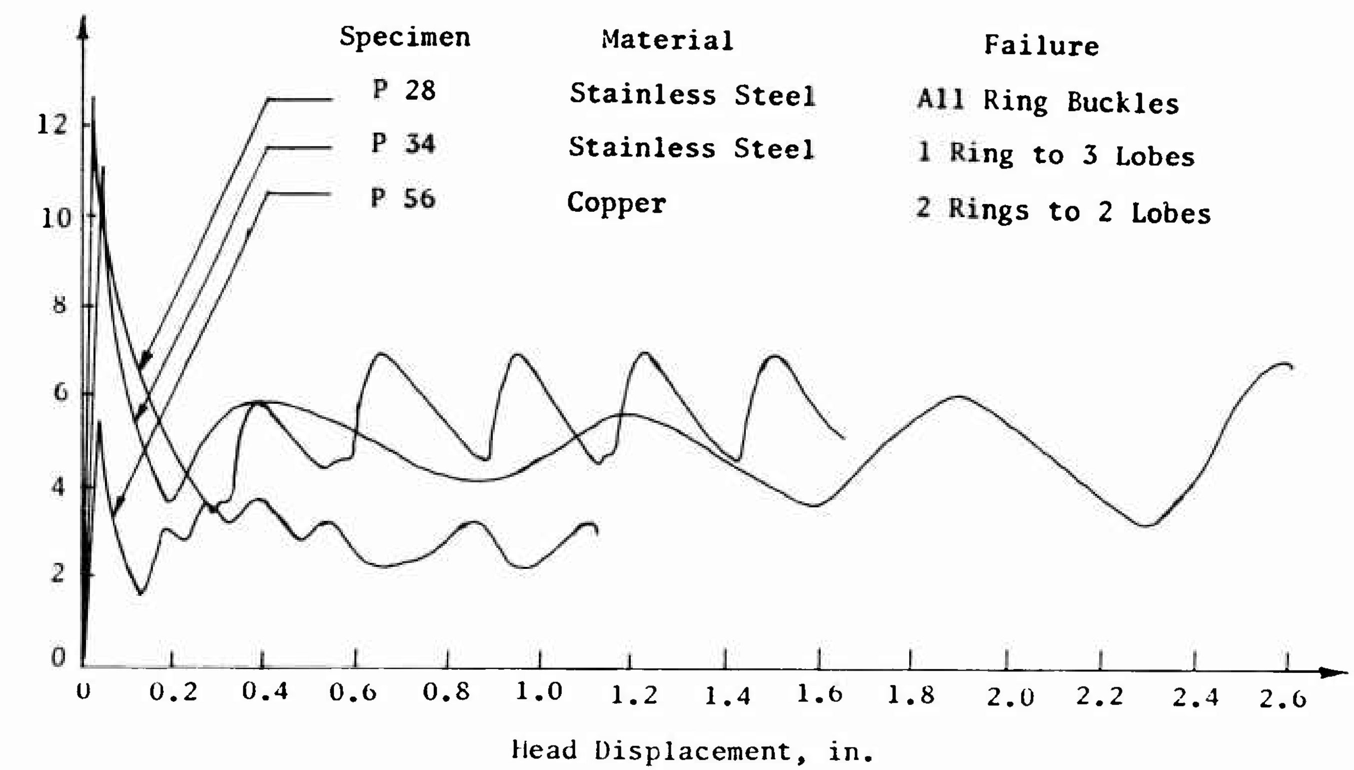


Figure 16. Load Deflection History for Symmetric and Nonsymmetric Buckle Pattern.

CONCLUSIONS

The experiments described in this report show that nonsymmetric buckle patterns occur in plastic buckling of thick-walled shells. These patterns are extremely regular in form and have a very remarkable similarity to the inextensional deformation patterns described by Yoshimura with reference to the limiting shapes which result from the elastic buckling of thin-walled cylindrical shells. It seems reasonable to suggest from the evidence available that, for a particular R, R/t, and material, there is a critical length at which there is a high probability of a compressed tube's changing its failing mode. Likewise, it does not seem unreasonable to propose that there are bounds to the values of R/t between which there is high probability of a consistent buckle number irrespective of the L/D. This observation is consistent with that of previous investigators.

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